A Brief History of Dislocation Theory

J. P. HIRTH

Early developments leading to the concept of a dislocation are discussed. The discoveries of 1934 are described. The subsequent evolution of various aspects of dislocation theory is outlined.

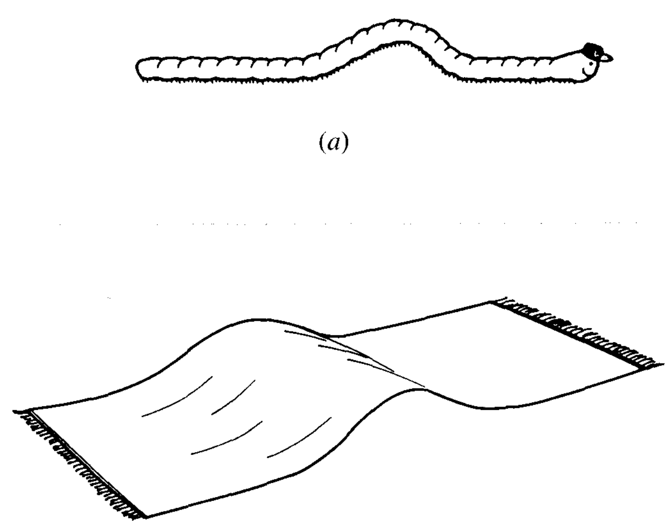
I. HARBINGERS AND ANTECEDENTS

WE are all now familiar with some of the pedagogical dislocation analogs, such as the crawling caterpillar and the puckered carpet, Figure 1. As documented in two valuable sources[1,2] for historical information, scientists in the 19th century had thought of a grain-type model of the aether containing localized defects, analogous to dislocations, that would enable the aether to deform.[3,4] Burton[3] called these defects *strain-figures*, and Larmor[5] described the creation of a strain-figure, that we would now recognize as a disc of twist boundary terminating within a material: he envisioned the creation of a lens-shaped cavity and the twisting of one surface in its plane, followed by the cementing together of the two sides. Somewhat related, a mosaic-block model of subgrains within a crystal was developed by Darwin[6] to explain the intensity of X-ray diffraction; the walls of these grains now being recognized as small-angle dislocation boundaries.

The elastic fields of dislocations in isotropic continua were derived beginning at the turn of the century. Weingarten[7] considered defects formed by the displacement of cut surfaces in bodies and showed that rigid displacements of the surfaces, creating dislocations, were required if strains in the body were to remain bounded. The stress fields of these defects were determined by Timpe[8] and the elastic properties were elaborated by Volterra,[9] who classified the general types of the defects into the six forms shown in Figure 2. Volterra called the defects *distorsioni*, but later Love,[10] who also contributed to the elastic theory, "ventured to call them dislocations." In the continuum mechanics literature there is still a tendency to follow this nomenclature and call all of these defects dislocations, but in the crystal plasticity area the defects in Figures 2(e), (f), and (g) are almost exclusively called disclinations while those in Figures (b), (c), and (d) are called dislocations.

The line of thought leading to the events of 1934 might be considered to have originated in the observations[11,12] in the 19th century that metals plastically sheared by forming slip bands or slip packets and in the work in X-rays in the early 20th century by M. VonLaue, P. P. Ewald, W. H. Bragg, and W. L. Bragg culminating in the concept of crystallinity.[13] A series of localized or extended defects was envisioned[14–19] in an effort to explain slip or fracture of crystals, some in view of the large discrepancy between theoretically predicted strengths for perfect crystals[20,21] and experimental results. One of these[16] had some elements of the Frenkel-Kontorova model described subsequently. Also of relevance to this model, Dehlinger[22] considered atoms above a slip plane to repose on a sinusoidal potential associated with atoms below, and conceptualized a localized defect, the *Verhakung*, that could cause the shear motion of atoms. In modern terms, the Verhakung is a pair of opposite sign edge dislocations separated by about an atomic distance.[23] In a model for the glide of single-crystals of zinc, Masing and Polanyi[24] proposed the configuration in Figure 3. Polanyi called the defects *verniers*[25] and the representation reflects this. The defects in the present context would be edge dislocations uniformly extended over about ten atomic distances, a configuration that is now known to be unrealistic; in essence Figure 3 represents a stack of elastically bent beams.[26] Yamaguchi[19] came very close to the concept of an edge dislocation; indeed his Figure 10 shows a double edge dislocation terminating a slip band that started at a surface, together with associated lattice strain. However, his view of the resistance to motion of the defect was tied to lattice curvature.

Thus, a number of models with some resemblance to dislocations had been postulated, but all missed some aspect of the true configuration, and the work in elasticity had not been related to crystal defects. One more early paper is relevant historically, although at first glance it might seem to be peripheral to the topic of dislocations. In their recollections,[26,27] both Orowan and Taylor particularly emphasize the importance of the theory of Griffith[28] for cracking of a



(a)

(b)

Fig. 1—Dislocation analogs used for pedagogical purposes: the caterpillar and the carpet.

J. P. HIRTH is Professor, Department of Metallurgical Engineering, The Ohio State University, Columbus, OH 43210.

This paper is based on a presentation made at the symposium "50th Anniversary of the Introduction of Dislocations" held at the fall meeting of the TMS-AIME in Detroit, Michigan in October 1984 under the TMS-AIME Mechanical Metallurgy and Physical Metallurgy Committees.

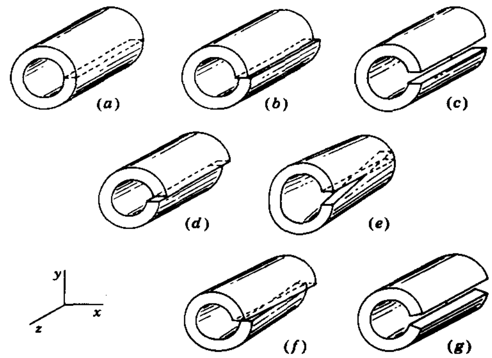


Fig. 2—The types of dislocations as classified by Volterra.[9]

brittle crystal. The theory led Orowan to think of slip originating at a crack tip as a consequence of stress concentration and led Taylor to envision the formation of a set of mode II shear microcracks in a slip band.

II. THE EVENTS OF 1934

This semicentennial seminar recognizes the three renowned 1934 papers[29,30,31] denoting the initiation of the dislocation theory of slip. As noted by Orowan,[26] Taylor submitted his paper first, while the other papers appeared first. A number of ideas concerning dislocations, including the mechanics of multiplication by "reflection" at a free surface, had been presented in Orowan's thesis of 1929.[26] The chain of events leading to his 1934 paper, as recounted by him, included the adventitious dropping of a zinc single crystal which led to observations of jerky flow and in turn to the consideration of models to explain the low resistance of a metal to shear. Orowan[29] sketched both the cross section of edge portions of a dislocation loop, showing the bent lattice planes resulting from the near-core strain field, and a schematic view of a dislocation loop on a glide plane, implicitly including edge and screw portions. Glide was accomplished by the growth of the loop in its glide plane. Polanyi had suggested that Orowan publish on his own,[26] but after discussion agreed to submit a paper to appear together with that of Orowan. Polanyi's sketch[30] of an edge dislocation in cross section reflects his earlier ideas on verniers, showing a uniformly extended dislocation. The model showed that dislocations could glide under stresses less than the theoretical shear stress, but did not reveal the local strain field near the dislocation. Polanyi had dropped his original term of vernier and called the defects *Versetzung,* a name also adopted by Orowan[25] and currently in use.

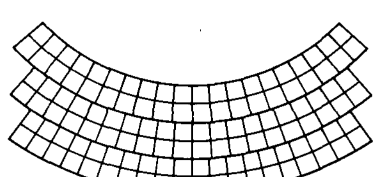


Fig. 3—Imperfections in a bent single crystal.[24]

In contrast to thinking of models for low shear strength, Taylor had been thinking of shear microcracks and of how these could lead to the strain-hardening of a material.[27] He realized that the ends of such a microcrack were dislocations that could move independently to produce glide. His drawings of an edge dislocation in cross section illustrated the glide process and showed the lattice distortion of the near-core strain field,[31] which he connected to the earlier elasticity calculations.[8,9,10] On the basis of the stress interactions, he also developed a two-dimensional model for work-hardening.[31] While he states that he regarded this model only as suggestive, it has many features of current models of hardening in uniaxial deformation,[32] as well as corresponding to a present-day conception of the dislocation wall structures in fatigued crystals.

Taylor continued to contribute broadly in the area of plasticity,[34] while Polanyi's interests turned to the field of social science. Orowan also contributed broadly, but more specifically performed other important work in dislocation theory. These contributions included his work on dislocation multiplication by the double-cross-slip mechanism;[35] on the Orowan mechanism of a dislocation bowing-out and bypassing hard particles in its glide path;[36] and on polygonization.[26] In addition he had developed the physical model that led to the calculation by Peierls of the Peierls stress.[37] Indeed, in an interesting recollection of his derivation of this stress, Peierls suggests that it might have been more appropriate to call it the Orowan stress.[38]

The 1934 papers clearly delineated the properties of edge dislocations. The equivalent properties for screw and mixed dislocations were described by Burgers[39] in the course of his work on the vector field theory for the elastic fields of dislocations, developed in analogy to vortices in hydrodynamics.

III. DISCLINATIONS AND DISPIRATIONS

As already mentioned, the elastic defects in Figures 2(e), (f), and (g) are now generally called disclinations. While isolated disclinations are unusual in metal crystals well below their melting points, they appear in a number of physical systems including the Abrikosov flux line lattice of superconductors, the Bloch wall lattice of ferromagnetic domains, and organic crystals, including liquid crystals.[40] They even appear, by geometric necessity, in geodesic domes.[41] For organic crystals, the recognition of disclinations preceded that of dislocations, with observations of disclinations in molecular crystals by Lehmann[42] at the turn of the century, and their physical description by Friedel.[43] The etymology of disclination is lengthy, the defects being called *Symmetriepunkte*,[42] *noyaux* or *fils*,[43] dislocations,[9,10] and other special names in noncrystalline analogs[40] (delta figures in fingerprints, for example). Frank used the term *Möbius crystals*[44] and later *disinclinations*.[45] However, according to Nabarro,[46] Frank later changed the name to *disclinations* after consulting with a philological colleague who said he was "disinclined" to use the former name. Nabarro first used the word disclination in print,[47] calling the defects in Figures 2(e) and (f) screw disclinations and that in Figure 2(g) an edge disclination. The terms wedge disclination[48] for Figure 2(g) and twist disclination[49] for Figures 2(e) and (f) were later suggested and are now used almost universally. Of historical interest, a twist disclination was described by Larmor.[5]

A line defect can have both dislocation and disclination character in an elastic continuum sense. The dislocation and disclination have discontinuities, respectively, in translational and rotational displacements. The combined defect has a screw symmetry displacement continuity, is unique in its crystallographic aspects, and is called a *dispiration*.[40,50]

The elastic properties of disclinations and dispirations have been extensively developed by deWit[51] and Chou.[50] Many aspects of the elastic and other properties of dislocations are treated in other papers presented at this symposium. Here, we briefly trace the historical development of various subfields of dislocation theory.

IV. GROWTH OF THE FIELD OF DISLOCATIONS

A. *Continuum Theory*

Brown,[52] while considering magnetic properties of dislocations, originated the concept of smearing discrete dislocations into a continuous array of infinitesimal dislocations. This method has resulted in connections with powerful methods of mathematics but describes properties of the *net* dislocation density and has some problems in uniqueness and the description of arrays of dislocations of opposite sign. In early work, Nye[53] described the connection between the net dislocation density tensor and the lattice curvature. Kondo[54] and Bilby, Bullough, and Smith[55] showed that the Cartan torsion of space is the continuum equivalent of the dislocation, with the Cartan circuit closely related to the Burgers circuit.[56] The latter authors used the continuum description to derive the geometric properties of grain boundaries. Kröner[57] developed the concept of the incompatibility, proportional to derivatives of the dislocation density, and descriptions of the elastic fields in terms of it. Further advances are discussed in several reviews.[57,58,59]

Another aspect of continuum models is the development of nonlocal or couple-stress theory, a method within continuum theory of treating nonlinear strains, in terms both of geometric plasticity effects and of the large elastic strain regions near dislocation cores. The theory originated with the concept of the Cosserat continuum.[58] The connection with the continuum theory of dislocations was clearly made in 1968[59] and is the subject of a recent book.[60] Nonlocal theory has been applied to discrete dislocation core regions as well.[61,62] Most recently, a gauge theory for dislocations has been presented,[63] but its connection to physical dislocations is obscure at present. Many of the concepts of continuum theory are lucidly discussed by Nabarro.[2]

B. *Elastic Theory*

The early calculations for elastic fields of dislocations[9,31] were corrected by Brown[52] and Koehler.[64] As mentioned previously, Burgers developed a vector field theory for dislocations leading to his renowned vector equation for the displacement field of a dislocation loop in terms of line integrals over its length and an area integral over its area of cut in its formation.[39] As reviewed elsewhere,[65] this equation led to the Peach-Koehler equation for the virtual thermodynamic force on a dislocation segment arising from external and internal stresses[66] and later to the Blin equation for the total elastic energy of a dislocation loop.[67] A complete treatment of early elastic theory of straight dislocations and simple shapes is given by Nabarro.[68] The concept of image forces on a dislocation associated with its interaction with free surfaces was enunciated by Eshelby[69] and Lothe.[70] Kroupa derived the elastic field of an infinitesimal dislocation loop, which could be integrated to give the elastic field of planar loops.[71]

Analytical solutions for the fields of complex curved dislocation configurations can be obtained if the dislocation lines are replaced by approximate shapes consisting of arrays of straight line segments. The fields of an angular pair was given by Yoffe;[72] that for single straight segments by Jøssang *et al.*[73] (corrected in both cases in Reference 65). Another single segment model was given by Eshelby and Laub,[74] in which the segment ends were connected by a continuous fan of infinitesimal dislocations. The Brown formula[75,76,77] gives the stress field of a single segment at a point in terms of the elastic energies of an infinite straight dislocation pair with lines passing through the point in question and the ends of the segment. This formula, which applies for isotropic or anisotropic elasticity, had its origins in Mura's equation, developed from the Burgers relation for displacements, for the displacement gradients in terms of line integrals.[78]

The anisotropic elastic theory for straight dislocations arose with the work of Eshelby *et al.*[79] Stroh[80] developed an alternate, explicit solution in terms of special vector functions, and Willis[81] presented an explicit solution employing Fourier analysis. The Stroh theory was elaborated as an integral theory, facilitating numerical calculations, by Barnett and Lothe.[82] Recent developments are discussed in several reviews.[65,83]

C. *Lattice Theory*

The original derivation of the Peierls stress[37] was corrected by Nabarro.[84] With the advent of fast computers, atomic calculations, replacing the glide-plane-strip-nonlinear region of the original work with a cylindrical core region centered on the dislocation, have been used to estimate the Peierls stress and energy. Early work is reviewed by Puls.[85] An important result was the finding that a dislocation is a center of dilatation producing a volume increase in a crystal of about an atomic volume per atomic plane cut by the dislocation; of this about 60 pct resides in the long-range strain field[86,87] and 40 pct in the highly nonlinear core region.[88] Closely related to the Peierls model are the Frenkel-Kontorova[89] one-dimensional model of spring connected balls on a periodic substrate with one less (more) ball than potential minima (see Reference 22) and the interface dislocations in oriented overgrowths.[90]

The presence of the Peierls barrier causes dislocations to tend to lie in low index directions at low temperatures. Where the dislocation locally leaves this direction a kink (in the glide plane) or jog is formed. These core defects can occur by geometric necessity, at thermal equilibrium, or by dislocation intersection.[91] They can serve as charged defects in ionic crystals,[92] important as extrinsic sources of charged point defects and of electronic defects.

Kink concepts were used to describe low-temperature deformation (creep and internal friction) by double-kink nucleation[93] and growth.[94,95] Interestingly, these models have been found to give kinetic equations that also apply to the recently developed theory for soliton motion in one dimensional conductors and to crystal growth by ledge motion.[65,96]

Jogs are important as sites where dislocation climb occurs,[97,98] the jogs acting as sites for local equilibration of vacancies (interstitials).[99]

D. *Groups of Dislocations*

The early theory for dislocation models of high angle boundaries was presented by Read and Shockley.[100] Their prediction for the energy of grain boundaries as a function of orientation is still applied.[101] The geometry of small-angle dislocation networks was advanced by Frank[102] who also related the boundary misorientation to the dislocation content of high angle boundaries. Elastic properties of cut and displaced surfaces corresponding to what could now be regarded as either arrays of glide dislocations on a slip plane or a core distribution of infinitesimal dislocations had been treated in early work by Somigliana.[103] The vector theory for possible dislocation sets in grain boundaries was evolved by Amelinckx and co-workers.[104] The concept of grain boundary dislocations (with Burgers lengths unequal to those of lattice dislocations and related to the grain boundary geometry) and their geometrical description was presented by Bollmann[105] and represents a topic of great current interest.[101]

The Frank-Read source was conceived independently by Frank and Read on the same day and when they realized this they decided to publish the idea jointly.[106] Other sources, most variants of the Frank-Read source and including spiral variants, were developed later, an important one widely observed experimentally[107] being the double-cross-slip mechanism.[35,108] The reflection mechanism had been postulated earlier,[26,109] but Leibfried's work[110] on the damping coefficient for phonon damped dislocation motion showed that it was not possible, a finding leading to the renewed thought on the subject by Frank. The equivalent of the Frank-Read source in climb, the Bardeen-Herring source was suggested later,[111] as was its spiral variant.

Work on dislocation pileups stems from the research of Eshelby *et al.*[112] A number of subsequent models are discussed by Chou and Li.[113] In the continuous infinitesimal dislocation approximation the results correspond exactly with those for a mode II continuum crack. Near-tip fields for nonplanar arrays can be taken directly from the equivalent continuum stress intensity factors in this approximation. Other arrays, including dipoles, multipoles, arrays of pileups, and intersecting segments can be treated by similar methods.[2,65,113]

E. *Partial Dislocations*

In close-packed crystals in particular, stacking faults have relatively small interfacial energies and dislocations can dissociate into partial dislocations. Nabarro[114] traces the stacking fault concept in hard ball stackings through R. Hooke, C. Huygens, W. H. Wollaston, and J. Kepler to a paper by Barlow.[115] The concept of partial dislocations was presented by Frenkel and Kontorova,[116] and later[91] for the fcc Shockley partial $\frac{1}{6}\langle 112\rangle$ and the fcc Frank partial[117] with Burgers vector $\frac{1}{3}\langle 111\rangle$. Thompson[118] introduced the convenient vector notation for fcc of the Thompson tetrahedron as well as the stair-rod partial, the smallest Burgers length form of which is $\frac{1}{6}\langle 110\rangle$. These concepts were elaborated to give more complex configurations including the Lomer-Cottrell lock, other locks, stacking fault tetrahedra, dislocation bends, dissociated dipoles, and dissociated jogs. Partial dislocation models for twinning were developed, an early one being the bcc pole mechanism of Cottrell and Bilby.[119]

With the advent of computer simulation, the core structure revealed other analogous defects. Dissociations over ~atomic distances, too small to correspond to well developed partial-stacking fault arrays, were observed.[120,121] The three-fold dissociation of bcc screw dislocations successfully explains the large Peierls stress for this case. The dissociated defects have some dislocation character and are called *fractional* dislocations.[122] The dissociation of the screw destroys the {110} mirror plane symmetry,[123] leading to additional core defects, where the symmetry switches, called *flips*.[65]

F. *Observations of Dislocations*

Discussions at conferences through the early 1950's contain numerous comments doubting the existence of dislocations. The final evidence removing all doubts by skeptics began with the discovery of the technique of direct observations of cell walls[124] and, subsequently, single dislocations[125,126] in transmission electron microscopy. This led to a still burgeoning field that identified many of the dislocation configurations discussed previously. Reviews of dislocation observations are given in two extensive articles by Amelinckx.[127,128] Other examples, some of which preceded the electron microscopy work, include: dislocation etch pits,[107] dislocations in bubble rafts,[129] screw dislocation growth spirals,[130] infra-red transmission,[131] and field-ion emission micrographs.[132]

Departing from crystal defects, we see dislocations all about us.[2,40] Examples in nature include dislocations in block wall lattices, superconducting flux line lattices, cellular eutectics, foam structures, brick walks, corn-cobs, the stripes on a zebra, the eye of a fly, the seeds in a sunflower, virus colonies, spider webs, and so forth.

V. CONCLUDING REMARKS

The foregoing compilation is not intended to be exhaustive, but to highlight particular advances in the understanding of dislocations, emphasizing early work. Other excellent work has been performed, some of which is discussed by others at this symposium. More extensive references are given for the subject of dislocations in the series of books listed in the Bibliography appended to the Reference list.

ACKNOWLEDGMENTS

This research was supported by the National Science Foundation under Grant DMR-8311620 and by a sabbatical leave at Stanford University granted by The Ohio State University.

REFERENCES

1. *The Sorby Centennial Symposium on the History of Metallurgy*, C. S. Smith, ed., Gordon and Breach, New York, NY, 1965.
2. F. R. N. Nabarro: *Theory of Crystal Dislocations*, Oxford Univ. Press, London, 1967.
3. C. V. Burton: *Phil. Mag.*, 1982, vol. 33, p. 191.
4. H. J. Axon and J. A. L. Matheson: *Nature*, 1956, vol. 178, p. 222, drew attention to the Rede Lecture of O. Reynolds in 1902 and reproduced his figures of defects in a two-dimensional, hexagonal, hard-ball model that we would now identify as a stairrod partial dislocation together with associated stacking faults. Indeed, C. S. Smith, Massachusetts Institute of Technology, personal communication, Aug., 1984, has noted that R. Descartes, in considering his corpuscular theory of the universe, showed defects resembling dislocations in such a two-dimensional array.
5. J. Larmor: *Phil. Trans. Roy. Soc. London*, 1897, vol. A190, p. 205.
6. C. G. Darwin: *Phil. Mag.*, 1914, vol. 27, pp. 315 and 675.
7. G. Weingarten: *Atti Accad. naz. Lincei Rc.*, 1901, vol. 10, p. 57.
8. A. Timpe: *Z. Math. Phys.*, 1905, vol. 52, p. 348.
9. V. Volterra: *Ann. Ecole Norm. Super. Paris*, 1907, vol. 24, p. 400.
10. A. E. H. Love: *A Treatise on the Mathematical Theory of Elasticity*, Cambridge University Press, Cambridge, 1920.
11. O. Mügge: *Neues Jahrb. Mineral.*, 1883, vol. I, p. 13.
12. A. Ewing and W. Rosenhain: *Phil. Trans. Roy. Soc. London*, 1899, vol. A193, p. 353.
13. An interesting historical account is given in W. L. Bragg: *The Crystalline State*, Bell, London, 1962, p. 268.
14. J. A. Ewing: *Rep. Brit. Assoc. Adv. Sci.*, London, 1907, p. 657.
15. W. Voigt: *Ann. Physik*, 1919, vol. 60, p. 638.
16. L. Prandtl: *Z. angew. Math. Mech.*, 1928, vol. 8, p. 85.
17. G. I. Taylor: *Trans. Faraday Soc.*, 1928, vol. 24, p. 121.
18. K. Yamaguchi: *Sci. Pap. Inst. Phys. Chem. Res. Tokyo*, 1929, vol. 11, p. 223.
19 A. Smekal: *Handb. d. Phys.*, 2nd ed., 1933, vol. 24, p. 795.
20. F. Zwicky: *Physikal. Z.*, 1923, vol. 24, p. 131.
21. J. Frenkel: *Z. Phys.*, 1926, vol. 37, p. 572.
22. U. Dehlinger: *Ann. Physik*, 1929, vol. 2, p. 749.
23. F. R. N. Nabarro: *Theory of Crystal Dislocations*, Oxford Univ. Press, London, 1967, p. 4.
24. G. Masing and M. Polanyi: *Ergebn. exakt. Naturw.*, 1923, vol. 2, p. 177.
25. E. Orowan: *The Sorby Centennial Symposium on the History of Metallurgy*, C. S. Smith, ed., Gordon and Breach, New York, NY, 1965, p. 360.
26. E. Orowan: *ibid.*, p. 359.
27. G. I. Taylor: *ibid.*, p. 355.
28. A. A. Griffith: *Phil. Trans. Roy. Soc. London*, 1920, vol. A221, p. 163.
29. E. Orowan: *Z. Phys.*, 1934, vol. 89, pp. 605, 614, and 634.
30. M. Polanyi: *Z. Phys.*, 1934, vol. 89, p. 660.
31. G. I. Taylor: *Proc. Roy. Soc. London*, 1934, vol. A145, p. 362.
32. For example see the reviews in A. W. Thompson, ed.: *Work Hardening in Tension and Fatigue*, TMS-AIME, Warrendale, PA, 1977.
33. D. Kuhlmann-Wilsdorf: *Mater. Sci. Eng.*, 1979, vol. 39, p. 127.
34. *Scientific papers of G. I. Taylor*, G. K. Batchelor, ed., Cambridge University Press, 1958-63, vols. 1-3.
35. E. Orowan: in *Dislocations in Metals*, M. Cohen, ed., AIME, New York, NY, 1954, p. 104.
36. E. Orowan: in *Symposium on Internal Stresses*, Inst. Metals, London, 1947, p. 451.
37. R. E. Peierls: *Proc. Phys. Soc.*, 1940, vol. 52, p. 23.
38. R. E. Peierls: in *Dislocation Dynamics*, A. R. Rosenfield, G. T. Hahn, A. L. Bement, and R. I. Jaffee, eds., McGraw-Hill, New York, NY, 1968, p. xiii.
39. J. M. Burgers: *Proc. Kon. Ned. Akad. Wetenschap.*, 1939, vol. 42, pp. 293, 378.
40. W. F. Harris: *Surface and Defect Prop. Solids*, 1974, vol. 3, p. 57.
41. R. W. Marks: *The Dymaxion World of Buckminster Fuller*, Reinhold, New York, NY, 1960.
42. O. Lehmann: *Flussige Kristalle*, Engelman, Leipzig, 1904.
43. G. Friedel: *Ann. Physique*, 1922, vol. 18, p. 273.
44. F. C. Frank: *Phil. Mag.*, 1951, vol. 42, p. 809.
45. F. C. Frank: *Disc. Faraday Soc.*, 1958, vol. 25, p. 19.
46. F. R. N. Nabarro: in *Fundamental Aspects of Dislocation Theory*, J. A. Simmons, R. deWit, and R. Bullough, eds., Spec. Publ. 317, Nat. Bur. Stand., Washington, DC, 1970, vol. 1, p. 710.
47. F. R. N. Nabarro: in *Int. Congress on Electron Diffraction and Cryst. Defects*, Pergamon, Oxford, 1966, p. II L-1.
48. K. H. Anthony, U. Essmann, A. Seeger, and H. Träuble: in *Mechanics of Generalized Continua*, E. Kröner, ed., Springer, Berlin, 1968, p. 355.
49. J. D. Eshelby, J. P. Hirth, and R. DeWit: in *Fundamental Aspects of Dislocation Theory*, J. A. Simmons, R. deWit, and R. Bullough, eds., Spec. Publ. 317, Nat. Bur. Stand., Washington, DC, 1970, vol. 1, p. 715.
50. T. W. Chou: in *Dislocation Modeling of Physical Systems*, M. F. Ashby, R. Bullough, C. S. Hartley, and J. P. Hirth, eds., Pergamon, Oxford, 1981, p. 405.
51. R. deWit: *J. Res. Nat. Bur. Stand.*, 1973, vol. 77A, pp. 49, 359; in *Dislocation Modeling of Physical Systems*, M. F. Ashby, R. Bullough, C. S. Hartley, and J. P. Hirth, eds., Pergamon, Oxford, 1981, p. 304.
52. W. F. Brown: *Phys. Rev.*, 1941, vol. 60, p. 139.
53. J. F. Nye: *Acta Metall.*, 1953, vol. 1, p. 153.
54. K. Kondo: *RAAG Memoirs of the Unifying Study of the Basic Problems in Engineering Sciences by Means of Geometry*, Gakujutsu Buuken Fukyu-Kai, Tokyo, 1955, vol. I, p. 453; also see Ref. 46, p. 761.
55. B. A. Bilby, R. Bullough, and E. Smith: *Proc. Roy. Soc. London*, 1955, vol. A231, p. 263.
56. E. Kröner: in *Dislocation Modeling of Physical Systems*, M. F. Ashby, R. Bullough, C. S. Hartley, and J. P. Hirth, eds., Pergamon, Oxford, 1981, p. 285.
57. E. Kröner: *Ergeb. angew. Math*, 1958, vol. 5, p. 1.
58. E. Cosserat and F. Cosserat: *Theorie des Corps Deformables*, Herman, Paris, 1909.
59. *Mechanics of Generalized Media*, E. Kröner, ed., Springer, Berlin, 1968.
60. I. A. Kunin: *Elastic Media with Microstructure*, Springer, Berlin, 1982.
61. A. C. Eringen: in *Nonlinear Equations in Phys. and Math.*, A. O. Barut, ed., Reidel, Dondrecht, 1978, p. 271.
62. A. G. Herrmann: in *Dislocation Modeling of Physical Systems*, M. F. Ashby, R. Bullough, C. S. Hartley, and J. P. Hirth, eds., Pergamon, Oxford, 1981, p. 110.
63. A. Kadic and D. G. Edelen: *A Gauge Theory of Dislocations and Disclinations*, Springer Lecture Notes in Physics, No. 174, Berlin, 1983.
64. J. S. Koehler: *Phys. Rev.*, 1941, vol. 60, p. 397.
65. J. P. Hirth and J. Lothe: *Theory of Dislocations*, 2nd ed., Wiley, New York, NY, 1982.
66. M. O. Peach and J. S. Koehler: *Phys. Rev.*, 1950, vol. 80, p. 436.
67. J. Blin: *Acta Metall.*, 1955, vol. 3, p. 199.
68. F. R. N. Nabarro: *Advan. Phys.*, 1952, vol. 1, p. 269.
69. J. D. Eshelby: *Phil. Trans. Roy Soc. London*, 1951, vol. A244, p. 87.
70. J. Lothe: in *Fundamental Aspects of Dislocation Theory*, J. A. Simmons, R. deWit, and R. Bullough, eds., Spec. Publ. 317, Nat. Bur. Stand., Washington, DC, 1970, vol. 1, p. 11.
71. F. Kroupa: *Czech. J. Phys.*, 1960, vol. 10B, p. 284.
72. E. H. Yoffe: *Phil. Mag.*, 1960, vol. 5, p. 161.
73. T. Jøssang, J. Lothe, and K. Skylstad: *Acta Metall.*, 1965, vol. 13, p. 271.
74. J. D. Eshelby and T. Laub: *Can. J. Phys.*, 1967, vol. 45, p. 887.
75. J. Lothe: *Phil. Mag.*, 1967, vol. 15, p. 353.
76. L. M. Brown: *Phil. Mag.*, 1967, vol. 15, p. 363.
77. V. L. Indenbom and S. S. Orlov: *Sov. Phys. Cryst.*, 1968, vol. 12, p. 849.
78. T. Mura: *Phil. Mag.*, 1963, vol. 8, p. 843.
79. J. D. Eshelby, W. T. Read, and W. Shockley: *Acta Metall.*, 1953, vol. 1, p. 251.
80. A. N. Stroh: *J. Math. Phys.*, 1962, vol. 41, p. 77.
81. J. R. Willis: *Phil. Mag.*, 1970, vol. 21, p. 931.
82. D. M. Barnett and J. Lothe: *Phys. Norvegica*, 1973, vol. 7, p. 13.
83. D. J. Bacon, D. M. Barnett, and R. O. Scattergood: *Prog. Mater. Sci.*, 1978, vol. 23, p. 51.
84. F. R. N. Nabarro: *Proc. Phys. Soc. London*, 1947, vol. 59, p. 256.
85. M. P. Puls: in *Dislocation Modeling of Physical Systems*, M. F. Ashby, R. Bullough, C. S. Hartley, and J. P. Hirth, eds., Pergamon, Oxford, 1981, p. 249.
86. C. Zener: *Trans. AIME*, 1942, vol. 147, p. 361.
87. R. A. Toupin and R. S. Rivlin: *J. Math. Phys.*, 1960, vol. 1, p. 8.

88. R. G. Hoagland, J. P. Hirth, and P. C. Gehlen: *Phil. Mag.*, 1976, vol. 34, p. 413.
89. J. Frenkel and T. Kontorova: *Phys. Z. Sowj.*, 1938, vol. 13, p. 1.
90. F. C. Frank and J. H. van der Merwe: *Proc. Roy. Soc. London*, 1949, vol. A198, p. 216.
91. R. D. Heidenreich and W. Shockley: *Report of Conf. on Strength of Solids*, Phys. Soc., London, 1948, p. 57.
92. F. Seitz: *Phys. Rev.*, 1950, vol. 80, p. 239.
93. J. Weertman: *Phys. Rev.*, 1956, vol. 101, p. 1429.
94. A. Seeger: *Phil. Mag.*, 1956, vol. 1, p. 651; A. Seeger and P. Schiller: *Acta Metall.*, 1962, vol. 10, p. 348.
95. J. Lothe and J. P. Hirth: *Phys. Rev.*, 1959, vol. 115, p. 543.
96. A. Seeger: *Z. f. Metallk.*, 1981, vol. 72, p. 369.
97. F. R. N. Nabarro: *Report of Conf. on Strength of Solids*, Phys. Soc., London, 1948, p. 75.
98. J. Bardeen: *Phys. Rev.*, 1949, vol. 76, p. 1403.
99. N. F. Mott: *Proc. Phys. Soc. London*, 1951, vol. B64, p. 729.
100. W. T. Read and W. Shockley: *Phys. Rev.*, 1950, vol. 78, p. 275.
101. See *Grain Boundary Structure and Kinetics*, Am. Soc. Metals, Metals Park, OH, 1980.
102. F. C. Frank: in *Report of the Conf. on Defects in Crystalline Solids*, Physical Society, London, 1955, p. 159; in *Report of the Symposium on the Plastic Deformation of Crystalline Solids*, Carnegie Inst. Tech., Pittsburgh, PA, 1950, p. 150.
103. C. Somigliana: *Atti. Accad. naz. Lincei Rc.*, 1912, vol. 23, p. 463; 1915, vol. 24, p. 655.
104. See S. Amelinckx and W. Dekeyser: *Solid State Phys.*, 1959, vol. 8, p. 325.
105. W. Bollmann: *Crystal Defects and Crystalline Interfaces*, Springer, Berlin, 1970.
106. F. C. Frank and W. T. Read: in Report of the Symposium on the Plastic Deformation of Crystalline Solids, Carnegie Inst. Tech., Pittsburgh, PA, 1950, p. 44; *Phys. Rev.*, 1950, vol. 79, p. 722.
107. W. G. Johnston and J. J. Gilman: *J. Appl. Phys.*, 1960, vol. 31, p. 632.
108. J. S. Koehler: *Phys. Rev.*, 1952, vol. 86, p. 52.
109. F. C. Frank: *Report of Conf. on Strength of Solids*, Phys. Soc., London, 1948, p. 46.
110. G. Leibfried: *Z. Phys.*, 1950, vol. 127, p. 344.
111. J. Bardeen and C. Herring: in *Imperfections in Nearly Perfect Crystals*, Wiley, New York, NY, 1952, p. 261.
112. J. D. Eshelby, F. C. Frank, and F. R. N. Nabarro: *Phil. Mag.*, 1951, vol. 42, p. 351.
113. Y. T. Chou and J. C. M. Li: in *Math. Theory of Dislocations*, T. Mura, ed., Am. Soc. Mech. Eng., New York, NY, 1969, p. 116.
114. F. R. N. Nabarro: *Theory of Crystal Dislocations*, Oxford Univ. Press, London, 1967, p. 199.
115. W. Barlow: *Nature*, 1883, vol. 29, pp. 186, 404.
116. J. Frenkel and T. Kontorova: *Fiz. Zh.*, 1939, vol. 1, p. 137.
117. F. C. Frank: *Proc. Phys. Soc. London*, 1949, vol. 62A, p. 202.
118. N. Thompson: *Proc. Phys. Soc. London*, 1953, vol. 66B, p. 481.
119. A. H. Cottrell and B. A. Bilby: *Phil. Mag.*, 1951, vol. 42, p. 573.
120. V. Vitek, R. C. Perrin, and D. K. Bowen: *Phil. Mag.*, 1970, vol. 21, p. 1049.
121. Z. S. Basinski, M. S. Duesbery, and R. Taylor: *Phil. Mag.*, 1970, vol. 21, p. 1201.
122. V. Vitek: *Cryst. Lattice Defects*, 1974, vol. 5, p. 1.
123. A. Seeger and C. Wüthrich: *Nuovo Cim.*, 1976, vol. 33B, p. 38.
124. R. D. Heidenreich: *J. Appl. Phys.*, 1949, vol. 20, p. 993.
125. P. B. Hirsch, R. W. Horne, and M. J. Whelan: *Phil. Mag.*, 1956, vol. 1, p. 677.
126. W. Bollmann: *Phys. Rev.*, 1956, vol. 103, p. 1588.
127. S. Amelinckx: *Solid State Phys.*, Suppl. 6, Acad. Press, New York, NY, 1964.
128. S. Amelinckx: in *Dislocations in Solids*, F. R. N. Nabarro, ed., North-Holland, Amsterdam, 1979, vol. 2, p. 67.
129. W. L. Bragg and J. F. Nye: *Proc. Roy. Soc. London*, 1947, vol. A190, p. 474.
130. F. C. Frank: *Disc. Faraday Soc.*, 1949, vol. 5, p. 48.
131. W. C. Dash: *J. Appl. Phys.*, 1956, vol. 27, p. 1193.
132. E. W. Müller: *Acta Metall.*, 1958, vol. 6, p. 620.

BIBLIOGRAPHY

1. W. T. Read, Jr.: *Dislocations in Crystals*, McGraw-Hill, New York, NY, 1953.
2. A. H. Cottrell: *Dislocations and Plastic Flow in Crystals*, Oxford Univ. Press, 1953.
3. J. Friedel: *Les Dislocations*, Gauthier-Villars, Paris, 1956; *Dislocations*, Addison-Wesley, Reading, MA, 1964.
4. E. Kröner: *Kontinuums theorie der Versetzungen und Eigenspannungen*, Springer, Berlin, 1958.
5. J. Weertman and J. R. Weertman: *Elementary Dislocation Theory*, Macmillan, New York, NY, 1964.
6. R. Bullough: *Dislocations*, Atomic Energy Res. Estab., Harwell, 1964.
7. D. Hull: *Introduction to Dislocations*, Pergamon, Oxford, 1965; D. J. Bacon and D. Hull: 3rd ed., in press, 1984.
8. F. R. N. Nabarro: *Theory of Crystal Dislocations*, Oxford Univ. Press, 1967.
9. J. P. Hirth and J. Lothe: *Theory of Dislocations*, McGraw-Hill, New York, NY, 1968; 2nd ed., Wiley, New York, NY, 1982.
10. T. Mura: *Mathematical Theory of Dislocations*, Am. Soc. Mech. Engin., New York, NY, 1969.
11. J. W. Steeds: *Anisotropic Elastic Theory of Dislocations*, Clarendon, Oxford, 1973.
12. R. W. Lardner: *Mathematical Theory of Dislocations and Fracture*, Univ. of Toronto press, 1974.
13. C. Teodosiu: *Modele Elastice ale Defectelor Cristaline*, Edit. Acad. Rep. Soc. Romania, Bucarest, 1977; *Elastic Models of Crystal Defects*, Springer, Berlin, 1982.
14. *Dislocations in Solids*, Vols. I-VI, F. R. N. Nabarro, ed., North-Holland, Amsterdam, 1979-84.
15. T. Mura: *Micromechanics of Defects in Solids*, Nijhoff, The Hague, 1982.
16. V. L. Indenbom and J. Lothe: *Elastic Strain Fields and Dislocation Mobility*, North Holland, Amsterdam, in preparation, 1984.